Market reaction to a bid-ask spread change: A power-law relaxation dynamics

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We study the relaxation dynamics of the bid-ask spread and of the midprice after a sudden variation of the spread in a double auction financial market. We find that the spread decays as a power law to its normal value. We measure the price reversion dynamics and the permanent impact, i.e., the long-time effect on price, of a generic event altering the spread and we find an approximately linear relation between immediate and permanent impact. We hypothesize that the power-law decay of the spread is a consequence of the strategic limit order placement of liquidity providers. We support this hypothesis by investigating several quantities, such as order placement rates and distribution of prices and times of submitted orders, which affect the decay of the spread.

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I. INTRODUCTION

Financial markets are complex systems where many agents interact by sending offers (limit orders) to buy or sell a given amount of a financial asset at a given price and/or market orders to buy or sell a given amount of a financial asset at the best available price. The limit and market orders and their time dynamics are stored in an information technology infrastructure called the limit order book of the market. The order book dynamics is investigated both within the field of market microstructure [1–18] and within the field of econophysics [19–28].

Many theoretical and empirical studies have examined microstructure properties of double auction financial markets. Limit order book data provide the maximum amount of information at the lowest aggregation level. Microstructure studies are important both for analyzing and modeling the dynamics of the limit order book and in the investigation of determinants of key financial variables such as the bid-ask spread (the difference between the best ask price and the best bid price simultaneously present in the market). Moreover, the statistical regularities observed in these investigations can provide empirical and modeling support or falsify conjectures about the origin of stylized facts observed in financial markets.

A large body of literature has been devoted to the modeling of trade and quote submissions in a financial market [1–28]. Models and empirical investigations consider a few classes of explanations for the existence of the spread: (i) inventory or liquidity effects which are faced by the market makers, i.e., traders acting as liquidity providers by placing limit orders;[1] (ii) market power (observed when a single market maker is in charge with a kind of monopolistic position); (iii) adverse selection, observed when a class of informed traders may have superior information on stock price which is not shared by market makers; and (iv) the order processing costs.

Different studies have considered various aspects of the problem and different conclusions have sometimes been reached manifesting the need for further model development and empirical investigation. For example, Huang and Stoll [12] concluded that order processing costs are often the largest determinant of the bid-ask spread, whereas Wyart *et al.* [27] concluded that the main determinant of the bid-ask spread is adverse selection. Another kind of investigation uses vector autoregressive models [4] to analyze and interpret the trade and quote price dynamics observed in a double auction financial market (with or without a specialist). Examples of this approach are given in Refs. [16,18], where the authors assess the role of the waiting time between consecutive transactions in the process of price formation.

The dynamics of an order book can be quite complex over time. Even for the most liquid stocks there can be substantial gaps in the order book, corresponding to a block of adjacent price levels containing no quotes. When such a gap exists next to the best price, a new market order can remove the best quote, triggering a large midquote price change. In fact, it has been shown that the distribution of large price changes is almost coincident with the distribution of gaps in the limit order book [29] in the high-frequency limit. The market order triggering the trade must have a size at least equal to the opposite best and can therefore be of small size. A market order producing a large price change also creates a temporary large spread. The opening of the spread might also reflect adverse selection related to the flux or alteration of the amount of "information" incorporated at each transaction [10,27]. The market then usually reverts the spread to a normal value as a consequence of the events immediately following the trade.

The presence of a large bid-ask spread poses challenging questions to various kinds of traders. When the spread is large liquidity takers, i.e., traders with a pronounced urgency to perform a market transaction, have a strong disincentive to

[1]Market makers sell at the ask price and buy back at the lower bid price. Their activity can be institutional or self-organized in fully electronic double auction markets.

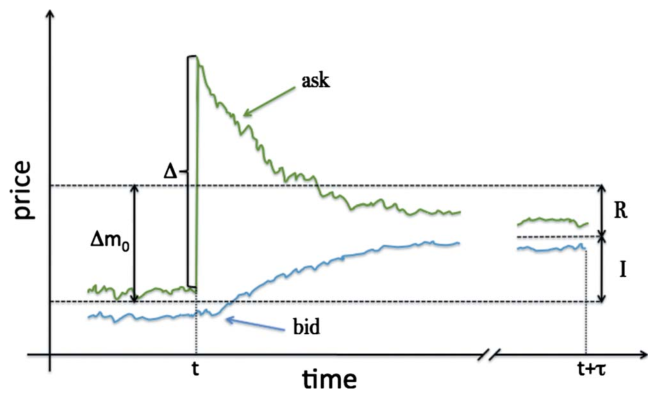

FIG. 1. (Color online) Schematic representation of the relaxation dynamics of the spread after an abrupt change. The (green) top and (blue) bottom curves represent the time series of the ask and the bid, respectively. At time t an event triggers a change Δ in the spread. The midprice changes by a quantity Δm_0. R and I indicate the price reversion and the permanent impact, respectively.

submit market orders given that the bid-ask spread is large. Conversely market makers trade by placing limit orders and therefore profit of a large spread by selling at the ask price and buying back at the lower bid price. Moreover there is a strong incentive to place limit orders in the spread given that a trader can attain the best position (price) in the book with the highest execution priority. However the optimal order placement inside the spread is a nontrivial problem. Slowly closing the spread by placing a limit order at a price just beating the best by one tick and waiting for a market order would give the best execution price (from the point of view of the trader placing the limit order), but this strategy risks being beaten by limit orders of other traders. After some time the price reaches a "normal" value attractive to market order submitters. The determination of the post reversion price value is an important problem for all types of traders. Liquidity providers have to decide how to readjust their quotes by taking into account the informed nature of the market order which generated the spread variation. On the other hand market impact is one of the most important costs of trading for liquidity takers. When a liquidity taker wishes to submit a large order she usually decides to split it in parts and trade it incrementally. Any transaction of a part of the large order pushes the price in a direction that makes the next transaction more unfavorable for her. Thus liquidity takers wish to minimize the price change due to their own trading and they need to know what the permanent part of their own trading is.

This paper empirically investigates how the market reacts to order book changes and how the spread and the price revert back to normal values. To fix the ideas consider the situation depicted in Fig. 1. The spread fluctuates around its typical value when at time t an event triggers an increase Δ of the spread. This paper investigates how spread and price revert back to their normal values after a long-time interval τ. Specifically, this paper focuses on two questions: (i) what is the typical dynamics of the spread between time t and time $t+\tau$ and how does the dynamics depend on Δ? (ii) What is the new level reached by the midprice at time $t+\tau$? More specifically, if the event at time t induces an immediate impact (i.e., a change) Δm_0 of the midprice (i.e., the mean value of the best bid and best ask price), what are the permanent impact I and the amount of price reversion R (see Fig. 1)?

Concerning question (i), we find empirical evidence of a scale-free power-law mean reversion to the normal level of the spread. We interpret the absence of a typical time scale in the reversion dynamics to a normal spread level as an indication of the existence of a strategic placement of limit and market orders submitted by the traders. Power-law relaxation has been, in fact, theoretically predicted and empirically observed in several complex systems where a multiplicity of time scales is intrinsically associated with the investigated system. Examples are spin glasses [30], microfracturing phenomena [31], internet dynamical responses [32], internet traffic [33], and price index dynamics after a financial market crash [34]. The hypothesis of strategic placement of limit and market orders is supported by analysis of the dependence of the rate of limit and market order submissions and cancellations from the distribution of the distance from the best price of the price of limit orders submitted inside the spread and from other indicators of the order book dynamics.

Concerning question (ii), by investigating both the ask and the midprice we empirically estimate the relation between a permanent market impact and the corresponding immediate market impact triggered by an order book change. Both for the ask and the midprice, we are able to observe an approximate linear relation between immediate and permanent impact.

Our paper is organized as follows. In Sec. II we present the data set used in our empirical analyses and a graphical representation methodology of the order book that might help to directly visualize various aspects of the book dynamics. Section III presents the power-law dynamics of the bid-ask spread observed after an order book update altering the spread. In Sec. IV we investigate the relation between the permanent impact and the immediate impact originated by an order book update affecting the ask or the midprice. In Sec. V we present empirical observations supporting the hypothesis that the power-law relaxation is due to the strategic placement of limit order submissions and cancellations. In Sec. VI we briefly discuss our results and draw some conclusions.

II. DATA AND THEIR GRAPHICAL REPRESENTATION

A. Data set

Our data set is composed of 71 highly capitalized stocks traded at the London Stock Exchange (LSE). The time period is the whole year 2002. The tickers of the investigated stocks are as follows: SHEL, VOD, GSK, RBS, BP., AZN, LLOY, REL, HSBA, BARC, HBOS, ULVR, BT.A, DGE, AV., PRU, BSY, WPP, RIO, ANL, TSCO, RTR, PSON, STAN, CBRY, BA., BG., BLT, BATS, NGT, AVZ, CPG, AAL, ARM, CNA, CW., RSA, KFI, SPW, SUY, IMT, RB., BZP, LGEN, ICI, MKS, GUS, SSE, DXNS, SHP, ALLD, OOM, BOG, BOC, HG., SCTN, BAA, LOG, RR, SMIN, HNS, GAA, NYA, SGE, WOS, AL., SFW, ISYS, III, BAY, and RTO. The order of the ticker symbol in the list is fixed by its rank when the stocks are sorted according to the size occupied by the stock in the database. SHEL occupied the largest amount of memory in the database, while RTO occupied the smallest memory among the considered stocks.

The LSE has a dual market structure consisting of a centralized limit order book market and a decentralized bilateral exchange. In London the centralized limit order book market is called the on-book market and the decentralized bilateral exchange is called the off-book market. In 2002 62% of LSE stock transactions occurred in the on-book exchange. In our study we consider only the on-book market. The on-book market is a fully automated electronic exchange. Market participants are able to view part of or the entire limit order book at any instant and to place trading orders and have them entered into the book executed or canceled almost instantaneously. The trading day begins and terminates with an auction. For this study, we ignore the opening and closing auctions and analyze only orders placed during the continuous auction period. Moreover, in order to avoid effects near the start and end of the day, we omit the first and last half an hour of trading from the calculation each day. That is we make a time series for each day from 8:30 a.m. to 4:00 p.m. and using it we calculate the conditional averages and the unconditional averages and repeat the process for each separate day, without including any time lags across different days. Finally, in most of our analyses, we removed the data of trading occurring on September 20, 2002. This is because on that day anomalous behavior of the spread due to unusual trading was observed as described below.

B. Graphical representation of the order book data

Most financial markets work through a limit order book mechanism. Agents can place market orders, which are requests to buy or sell a given number of shares immediately at the best available price or limit orders which also state a limit price, corresponding to the worst allowable price for the transaction. Limit orders often fail to result in an immediate transaction and are stored in a queue called the *limit order book*. At any given time t there is a best (lowest) offer to sell with price $a(t)$ and a best (highest) bid to buy with price $b(t)$. These are also called the *ask* and the *bid*, respectively. The difference $s(t)=a(t)-b(t)$ between the ask and the bid is called the *spread*. The *midprice* is defined as $m(t)=[a(t)+b(t)]/2$. The difference between the best buy price and the second best buy price is called the first buy gap, whereas the difference between the second best sell price and the best sell price is called the first sell gap. Gaps provide a proxy for the immediate liquidity present in the limit order book.

Visualizing the dynamics of the limit order book is a complex task because many orders are present in the book at a given time. We represent the book dynamics with a graph of the type shown in Fig. 2 for the stock Astrazeneca (AZN) in two representative days. The top panel shows an example of a normal trading period recorded on September 4, 2002, whereas the bottom panel shows an unusual day, specifically a period of September 20, 2002, when an unusual rogue trading pattern was occurring. In these plots each line shows a price level. Price levels appear as the result of orders being submitted into the book. Similarly price levels may disappear due to cancellations of orders or due to trades. Of course there may be other orders submitted onto existing price levels, but these are not explicitly shown in the plots. The ask is shown as a green line bounding from above the gray area of the bid-ask region and the bid is shown as a blue line binding from below the bid-ask region. The first sell gap is the block of unoccupied price levels above the ask before the next sell price level and the first buy gap is the block of unoccupied price levels below the bid before the next buy price level. Indeed the difference between the two figures, although both are AZN just a few trading days apart, illustrates the heterogeneity of order book dynamics.

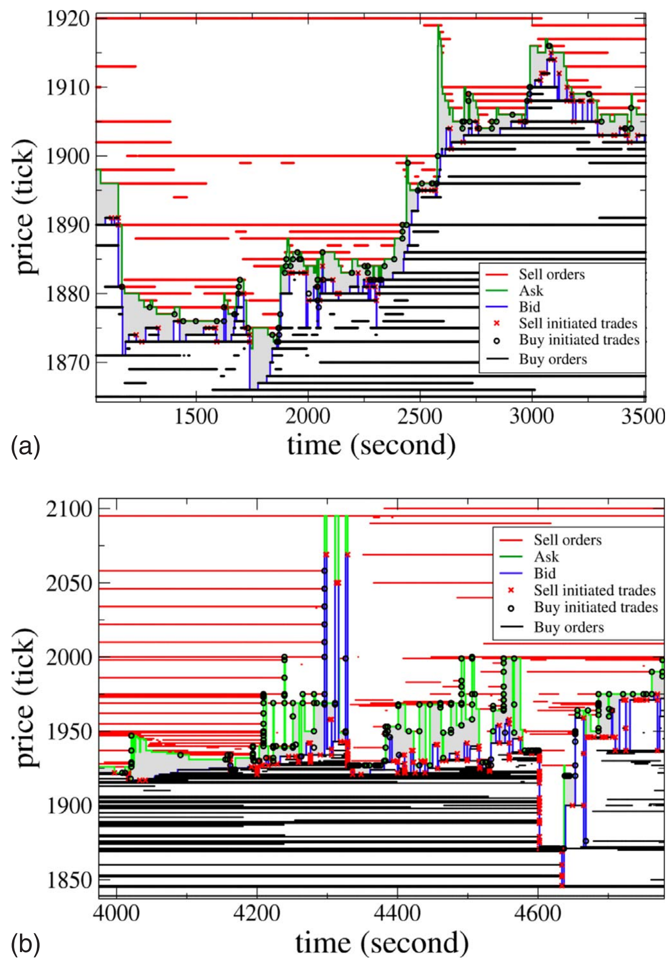

FIG. 2. (Color online) Examples of limit order book time series. Top panel: AZN order book dynamics on September 4, 2002 during a normal trading day. Time is given in seconds from 8:00 a.m. Bottom panel: AZN order book dynamics on September 20, 2002 when a rogue trading pattern was occurring. Time is given in seconds from 9:00 a.m. The gray region indicates the bid-ask interval. This interval is bounded above by the best ask (green bounding line) and below by the best bid (blue bounding line). Above the bid-ask region (red) horizontal lines indicate sell limit orders, whereas below the bid-ask region (black) horizontal lines indicate buy limit orders. Seller initiated transactions are shown as (red) crosses and buyer initiated transactions are shown as (black) circles.

The September 4 trading is normal and representative of other trading days. Here price drifting is visible by a tick by tick order submission process, as well as some large fluctuations. Large fluctuations occur when there is a large first gap and a trade (or sometimes cancellation) removes all the quotes at the best price, such as can be seen around $t \approx 2500$. This large fluctuation creates a large spread. The spread relaxes generally with a slow dynamics to a more normal value in part due to tick by tick "price beating" order submissions into the spread. Such price beating can occur on the same side of the book as the trade which removed the best and created the large spread, in which case the action is to revert the midprice. Alternatively it can occur on the opposite side, in which case the action is to produce midprice drifting.

In the example from September 20 many exceptional aggressive market orders are submitted. These orders cannot be filled solely by orders at the best so they cut across several price levels in the book, creating a highly volatile spread dynamics. The spread can become huge, of the order of a hundred ticks, and large midprice fluctuations result. It should be noted that the scales of price axes are quite different in the two panels of Fig. 2. In the top panel of Fig. 2 the y axis covers slightly more than 50 ticks, whereas the same axis covers more than 200 ticks in the bottom panel. It should be noted that this behavior is absolutely unusual in the sense that the aggressiveness of some market orders detected during September 20 is not observed in almost any other order submitted during the entire calendar year of 2002. Indeed, in 2002 we observed this kind of market order submission only during this particular day. For this reason, we have removed this specific day in all our analyses.

The order book dynamics presents fluctuations in order submission rates, cancellation rates, and trade rates which depend on the spread and size of preceding price fluctuations. All these fluctuations produce a nontrivial price-time coupling and "cascade" like dynamics. When the order book is plotted as in Fig. 2 some complex patterns of order book dynamics become evident. One particular example is the time asymmetry created by the spread dynamics, whereby the spread opens by few large fluctuations and closes by many small ones. When one studies the midprice or return time series this time asymmetry is not apparent as in the direct visualization obtained by the kind of plot presented in Fig. 2. This type of plot is an extension of the plots originally presented in [7]. However, different from previous plots, the present version contains full information about the status of the order book. This kind of figure can be a useful tool for the investigation of the order book dynamics during days when anomalous trading behavior is present. Moreover, a direct investigation of the bottom panel of Fig. 2 shows that this graphical tool can also be useful for distinguishing different types of high-frequency trading patterns.

III. CONDITIONAL SPREAD RELAXATION

The time series describing the dynamics of the spread is characterized by two stylized facts (see, for example, [25]). First, the unconditional spread distribution has a density

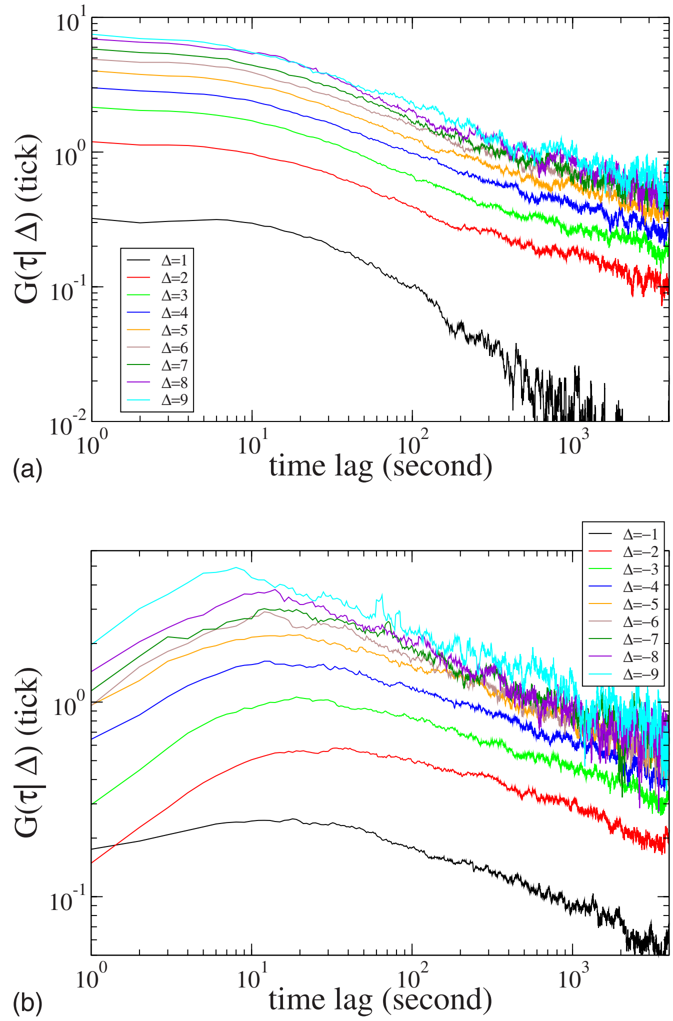

FIG. 3. (Color online) Conditional spread decay $G(\tau|\Delta)$ defined in Eq. (1) for the stock AZN. Top panel shows $G(\tau|\Delta)$ for different positive values of Δ (in ticks) corresponding to an opening of the spread at time lag $\tau=0$. Different curves refer to different values of Δ varying from 1 to 9 from bottom to top, respectively. Bottom panel shows $G(\tau|\Delta)$ for different negative values of Δ (in ticks) corresponding to a closing of the spread at time lag $\tau=0$. Different curves refer to different values of Δ varying from −1 to −9 from bottom to top, respectively.

function with a fat tail. Some papers suggest that the tail of the spread distribution is well fitted by a power-law function [25,26], whereas in other studies the spread seems to have an exponential tail [27]. The second fact is that spread seems to be described by a long memory process [25]. This implies that the autocorrelation of the spread decays asymptotically as a power law with an exponent smaller than 1. The long memory property of the spread may be related to the recently suggested proportionality between spread and volatility per trade [27]. Since volatility is a long memory process, spread is also a long memory process. The spread is a mean reverting process. Market orders and cancellations at the best can increase the spread whereas limit orders in the spread decrease the spread.

In this paper we are mainly interested in the conditional dynamics of the spread. We wish to characterize the dynam-

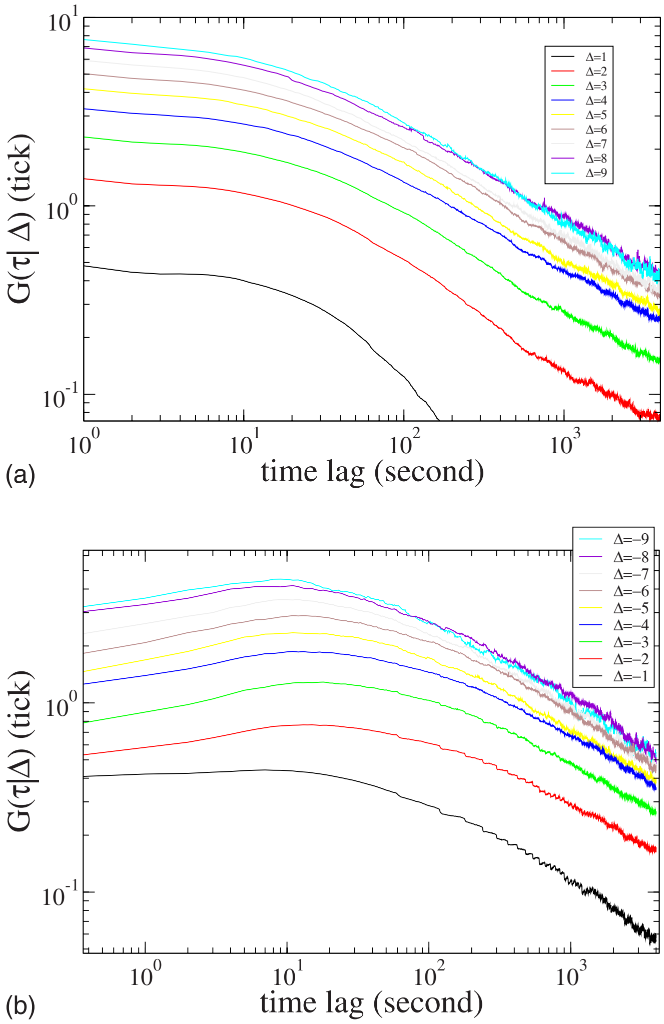

FIG. 4. (Color online) Conditional spread decay $G(\tau|\Delta)$ defined in Eq. (1). The curves are obtained by averaging $G(\tau|\Delta)$ over the 71 stocks of our sample. Top panel shows $G(\tau|\Delta)$ for different positive values of Δ (in ticks) corresponding to a opening of the spread at time lag $\tau=0$. Different curves refer to different values of Δ varying from 1 to 9 from bottom to top, respectively. Bottom panel shows $G(\tau|\Delta)$ for different negative values of Δ (in ticks) corresponding to a closing of the spread at time lag $\tau=0$. Different curves refer to different values of Δ varying from −1 to −9 from bottom to top, respectively.

ics of the spread $s(t)$ conditional to a spread *variation*. In other words, we wish to answer the following question: how does the spread return to a normal value after a spread variation? To this end we compute the quantity

$$G(\tau|\Delta) = E[s(t+\tau)|s(t) - s(t-1) = \Delta] - E[s(t)]. \tag{1}$$

Here and in the following $E[\cdots]$ indicates an average over time t. Figure 3 shows this quantity for the stock AZN as a function of τ for different positive and negative values of Δ. The decay of $G(\tau|\Delta)$ as a function of τ is very slow and for large values of τ is compatible with a power-law decay. The decay observed for AZN is representative of the decay observed for many investigated stocks. In order to obtain better statistics in Fig. 4 we plot $G(\tau|\Delta)$ averaged over the 71 stocks of our sample. We are aware that aggregating data from different stocks can create biases and/or spurious statistical effects in the estimation process. However the comparison of the averaged data with the data from different individual stocks suggests that the power-law decay of the spread is a common behavior to many stocks.

As in the individual case the asymptotic decay is compatible with a power law, $G(\tau|\Delta) \sim \tau^{-\delta}$, and the fitted exponent δ is around 0.4–0.5. The slow decay of the spread indicates that large changes in the spread are reverted to a normal value with a very slow dynamics. The power-law fit suggests that there is not a typical scale for the spread decay. A similar slow decay of the spread was observed by Zawadowski *et al.* in Refs. [22,23]. The main difference with this work is that Zawadowski *et al.* investigated the decay of the spread conditional on a negative change in the midprice rather than in the spread itself. Moreover, the investigated market and database are quite different. Zawadowski *et al.* investigated the New York Stock Exchange (NYSE) and the National Association of Securities Dealers Automated Quotations (NASDAQ) by using the Trade and Quote (TAQ) database. They found a slow decay of the spread at NYSE but not at NASDAQ. This last result does not seem to be consistent with our findings especially when considering the fact that the LSE is probably closer to NASDAQ than to NYSE due to the presence of the specialist at NYSE. In fact, both at LSE and NASDAQ most of the trading is performed through a fully electronic double auction mechanism.

It is worth noting that the slow spread decay is not a consequence of the long memory property of the spread itself. To show why this is the case, let us consider a generic stationary stochastic process $x(t)$. The response function $G(\tau|\Delta)$ is the expectation value $E[x(t+\tau)|x(t)-x(t-1) = y(t) = \Delta] - E(x)$. In general,

$$\begin{aligned} E[x(t+\tau)y(t)] &= \int p[x(t+\tau),y(t)]x(t+\tau)y(t)dxdy \\ &= \int dy\left\{\int xp[x(t+\tau)|y(t)]dx\right\}p(y)y \\ &= \int y(t)E[x(t+\tau)|y(t)]p(y)dy, \end{aligned} \tag{2}$$

therefore

$$\begin{aligned} E[x(t+\tau)y(t)] - E(x)E(y) = \int \{&E[x(t+\tau)|y(t)] \\ &- E(x)\}p(y)y(t)dy, \end{aligned} \tag{3}$$

i.e., the lagged covariance between x and y is the weighted average of the response functions $G(\tau|\Delta)$ with weights given by $\Delta p(\Delta)$. If the τ dependence of $G(\tau|\Delta)$ is the same for all values of Δ, then also the lagged covariance $E[x(t+\tau)y(t)] - E(x)E(y)$ has the same τ dependence. In this case the lagged covariance $E[x(t+\tau)(x(t)-x(t-1))] - E(x)E(x(t)-x(t-1)) = E[x(t+\tau)(x(t)-x(t-1))]$ is also equal to $\rho(\tau)-\rho(\tau-1)$, where $\rho(\tau) = E[x(t+\tau)x(t)] - [E(x)]^2$ is the autocovariance of $x(t)$. Suppose that $x(t)$ is a long memory process, i.e., that $\rho(\tau) \sim A\tau^{-\beta}$ with $0<\beta<1$. Then our argument shows that

$$E\{x(t+\tau)[x(t)-x(t-1)]\} = \frac{A}{\tau^\beta} - \frac{A}{(\tau+1)^\beta} \sim \frac{A\beta}{\tau^{\beta+1}}, \quad (4)$$

where we have approximated the result for large values of τ. This result shows that by assuming the spread is a long memory process with an autocorrelation function decaying as $\tau^{-\beta}$ with $\beta<1$ and that $G(\tau|\Delta)$ has the same τ dependence for all Δ, one should expect that $G(\tau|\Delta)$ decays asymptotically as a power law but with an exponent $1+\beta$ larger than 1. Our empirical results shown in Fig. 3 show that $G(\tau|\Delta>1) \sim \tau^{-0.4}$, while $G(\tau|\Delta=1) \sim \tau^{-0.9}$. Even if the exponent of the decay of the response function is different for $\Delta=1$ and $\Delta>1$, both exponents are smaller than 1.

The argument above suggests that the slow decay of $G(\tau|\Delta)$ cannot be simply explained by the long memory property of the spread. We can also rule out that the observed slow decay is an artifact of the intraday pattern of the spread. In fact, we have observed that the intraday average profile of the spread shows a maximum value at the market opening (8.00 a.m.) and after that it is on average monotonically decreasing to an almost constant value, which is observed from approximately 10.00 a.m. until the market closure (4.30 p.m.). We have verified that the intraday pattern of the spread does not play a role by repeating our analysis for the smaller time window elapsing from 10.00 a.m. to 4.00 p.m. when the average spread is approximately constant. When we repeat our investigation by conditioning the time within the cited time window we observe a conditional spread decay characterized by the same power-law functional form shown in Figs. 3 and 4.

In conclusion, the observed power-law slow decay of $G(\tau|\Delta)$ is not a consequence of the long memory property of the spread or of the intraday pattern of the spread. The power-law decay is also not an artifact due to the aggregation of the response of several stocks with different values of tick (in our data set the tick value ranges from 0.25 to 1 pence [35]) or different values of the mean spread [the mean value of the spread ranges from 1.2 to 4.7 ticks approximately in the investigated stocks (see Fig. 8 below for detailed value for each stock)]. In fact, we have pointed out above that the power-law decay observed for AZN is representative of the behavior observed for many investigated stocks. We observe the power-law decay of the conditional spread not only at an aggregate level but also at the level of individual stocks for many stocks. Finally, it is worth noting that even zero intelligence models are unable to explain the spread reversion dynamics [36]. For all these reasons, we postulate that the power law slow decay of the conditional spread may be due to the strategic behavior of the traders and to the mechanism through which the order flow closes the spread. In the following sections we give some empirical support to this hypothesis.

IV. PRICE REVERSION AND PERMANENT IMPACT AFTER A PRICE CHANGE

In this section we consider the permanent impact of a price fluctuation. The bid or ask can fluctuate in several ways. First a limit order can be submitted into the spread, second a cancellation can remove the best, and third a trade can remove the best. The second possibility is rather rare because there are usually several orders at the best owned by different trading agents and they all have to be canceled independently. On the other hand a single submitted market order can remove all the orders at the best price in one trade. Indeed when a market order arrives in the market, it may trigger a trade which creates a price change. This immediate price change is the immediate impact. The properties of immediate impact as a function of the trading volume and of the market capitalization of the stock have been studied, for example, in [37–41]. The transaction and the consequent price change generate a cascade of events in reaction. After a sufficiently large period of time the effect of the trade has vanished and the price will be, in general, different from the price before the trade. The variation in price is the permanent component of the impact of the trade.

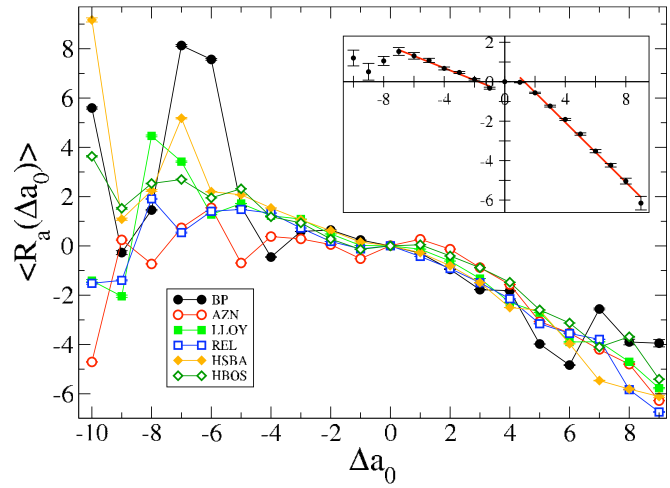

FIG. 5. (Color online) Value of $\langle R_a(\Delta a_0)\rangle$ for the ask of six highly traded and representative stocks versus Δa_0. Standard errors are within symbol size. The stocks are BP. (black filled circle), AZN (red open circle), LLOY (light green filled square), REL (blue open square), HSBA (orange filled diamond), and HBOS (green open diamond). The inset shows the considered quantity averaged across the first 55 stocks of our database, and here the error bars show the stock ensemble average standard errors. Linear regression fits to the points $\Delta a_0=1,\ldots,9$ and $\Delta a_0=-1,\ldots,-7$ are shown as (red) solid lines. For the positive Δa_0 range we find $\langle R_a(\Delta a_0)\rangle=0.98-0.76\Delta a_0$, while for the negative range we obtain $\langle R_a(\Delta a_0)\rangle=-0.53-0.3\Delta a_0$.

In this study, different from previous ones, we are interested in measuring the permanent impact of an order book update altering the price of an asset. We refer to this concept as *fluctuation impact*. With this term we refer to the impact on the price conditional to a price fluctuation (caused by any type of event) happening at a previous time. Since there are different prices in the market at a given time (bid, ask, midprice, etc.), we consider the impact on the ask and on the midprice.

As mentioned above fluctuation impact is the impact on the price conditional to a price fluctuation at a previous time. We consider first the permanent impact of ask price fluctuations. Consider the events happening at time t. The ask price changes due to these events by a quantity $\Delta a_0 = a(t) - a(t-1)$, where $a(t)$ is the ask price at the end of second t. Δa_0 is the immediate impact. After a sufficiently large time lag τ the ask price will be $a(t+\tau)$ and the permanent impact is $I_a(\tau) = a(t+\tau) - a(t-1)$. Thus the permanent impact can be decomposed as

$$I_a(\tau) = \Delta a_0 + R_a(\tau), \tag{5}$$

where $R_a(\tau) = a(t+\tau) - a(t)$ is the price change due to order submission and other events happening after the trade at time t has been completed. R_a is the price reversion after the event at time t. We measure first the conditional quantity

$$\widetilde{E}[R_a(\tau)|\Delta a_0] \equiv E[a(t+\tau) - a(t)|\Delta a_0] - E[a(t+\tau) - a(t)], \tag{6}$$

where we subtract the unconditional mean $E[a(t+\tau) - a(t)]$ in order to avoid spurious effects due to the finiteness of the time series. If we let $\tau \to \infty$ then we obtain the permanent impact. Since we cannot take the limit $\tau \to \infty$ in the calculation of the permanent impact, we use as a proxy

$$\langle R_a(\Delta a_0)\rangle = \frac{1}{T}\sum_{\tau=t_1}^{t_1+T} \widetilde{E}[R_a(\tau)|\Delta a_0], \tag{7}$$

where with the symbol $\langle \cdots \rangle$ we indicate a time and ensemble average for each value of Δa_0 and we take $t_1 = 500$ s and $T = 500$ s in the time average. In Fig. 5 we show $\langle R_a(\Delta a_0)\rangle$ for some highly traded and representative stock. There are large variations across stocks, some showing trend following for small positive Δa_0 and all showing partial reversion for large positive Δa_0. As described above, positive Δa_0 correspond to trade (or cancellation) initiated fluctuations. We note that one tick positive immediate fluctuation in AZN $\Delta a_0 = 1$ induces on average a further 0.25 tick positive fluctuation in the long-time average $\langle R_a(\Delta a_0)\rangle$. This shows the presence of a trending phase of the ask price in some stocks, which reinforces the direction of the price change. In other stocks, for example BP., one tick positive ask change induces a negative fluctuation of $\langle R_a(\Delta a_0)\rangle$ The inset of Fig. 5 shows the average behavior of $\langle R_a(\Delta a_0)\rangle$ across 55 stocks. A clear linear behavior of $\langle R_a(\Delta a_0)\rangle$ as a function of Δa_0 can be seen in different Δa_0 intervals. Also there is a significant asymmetry in $\langle R_a(\Delta a_0)\rangle$ for positive and negative Δa_0 since their permanent impact behavior is quite different. In particular for the range $\Delta a_0 = 2-9$ ticks the points lie on a line of slope approximately $-3/4$, while the $\Delta a_0 = 1$ point is close to zero. In other words, large positive spread fluctuations are partially reverted, while positive one tick spread opening fluctuations are balanced on the long run. The reverting behavior after a large positive ask fluctuation is related to the decay of the large spread opened up by the fluctuation discussed above. This shows that positive ask fluctuations have both a Δa_0 independent part and a part which depends on Δa_0. The total fluctuation composed of the initial fluctuation and of the successive part is $\langle I_a(\Delta a_0)\rangle \sim \alpha_a + \beta_a \Delta a_0$, where α_a is roughly one tick and β_a is approximately 1/4 ticks. Negative ask fluctuations, $\Delta a_0 = -1$, i.e., orders just beating the ask by one tick, are likely to be followed by further sell orders falling in the spread since they have a trend following permanent impact of approximately 1/2 tick. This is again the price beating behavior described above. Larger spread closing fluctuations, negative Δa_0, show a negative slope for the range -2 to -7. This means that spread closing fluctuations in this range are themselves reverted. This may be because the equilibrium spread is, in general, not the possible minimum one tick but maybe several ticks, and smaller spreads than equilibrium may be created by orders falling into the spread which must then revert to equilibrium.

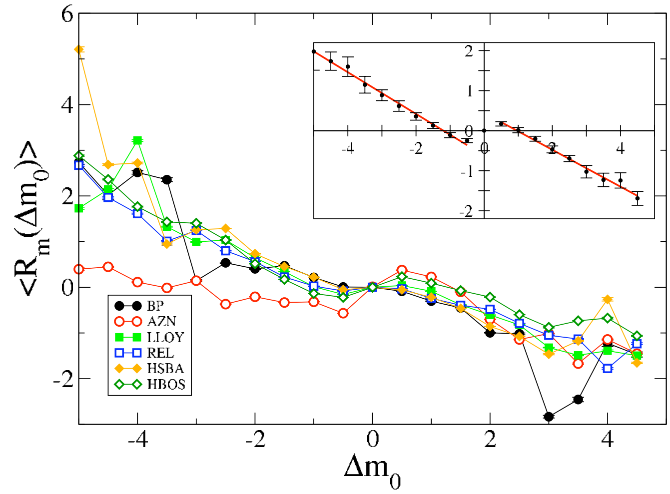

FIG. 6. (Color online) Value of $\langle R_m(\Delta m_0)\rangle$ for the midprice of six highly capitalized stocks. Standard errors are within symbol size. The stocks and the corresponding symbols are the same as in Fig. 5. The inset shows $\langle R_m(\Delta m_0)\rangle$ averaged across 55 stocks. In this case the error bars show the stock ensemble average standard errors. Solid (red) lines are linear fits.

We now consider the fluctuation impact on the midprice. Midprice permanent impact is defined analogously to the ask price permanent impact as $\widetilde{E}[R_m(\tau)|\Delta m_0] \equiv E[m(t+\tau) - m(t)|\Delta m_0] - E[m(t+\tau) - m(t)]$, where $m(t)$ is the midprice at time t and $\Delta m_0 = m(t) - m(t-1)$ is the midprice immediate impact. Again we let $\tau \to \infty$ to obtain the permanent impact

$$\langle R_m(\Delta m_0)\rangle = \frac{1}{T}\sum_{\tau=t_1}^{t_1+T} \widetilde{E}[R_m(\tau)|\Delta m_0]. \tag{8}$$

In Fig. 6 we show $\langle R_m(\Delta m_0)\rangle$. As for the ask $\langle R_m(\Delta m_0)\rangle$ is significantly different from zero independently of Δm_0. Some stocks show quite strong trend following effects for small values of Δm_0 with quite strong reversion effects for large values of Δm_0. For example, a half tick positive immediate fluctuation in AZN, $\Delta m_0 = 1/2$, induces on average a further half tick positive fluctuation in the long-time average, $\langle R_m(\Delta m_0)\rangle$, and furthermore a half tick negative immediate fluctuation can induce a further half tick negative permanent change. On the other hand, larger immediate fluctuations are followed by changes in the opposite sign, i.e., partial reversion. There are however quite strong variations across stocks—for example, HSBA does not seem to show the strong trend following behavior for small Δm_0 seen in AZN

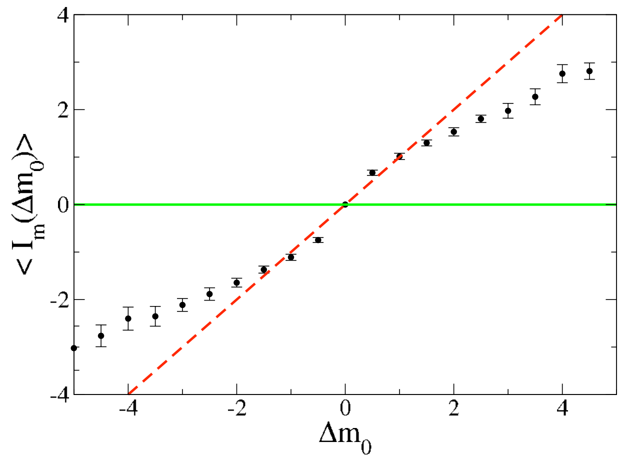

FIG. 7. (Color online) Value of $\langle I_m(\Delta m_0)\rangle$ averaged across the 55 stocks. The error bars show the stock ensemble average standard errors. The (red) dashed line is the line $y=x$ that would be obtained in case of a permanent impact equal to the immediate impact Δm_0. The (green) horizontal line is the line $y=0$ that would obtained in case of zero permanent impact, i.e., complete reversion.

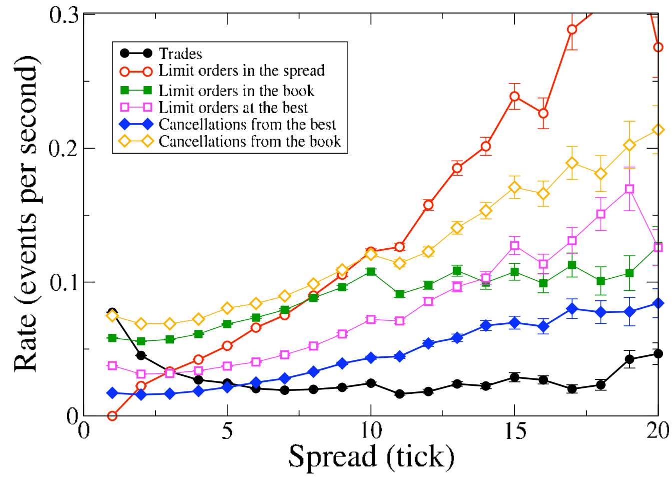

FIG. 9. (Color online) Rates of a series of events in the limit order book conditional to the size of the spread. The rate of transactions is shown as a black filled circle. Limit orders are divided into limit orders in the spread (red open circle), limit orders at an existing bid or ask best (purple open square), and limit orders placed inside the book (green filled square). Similarly cancellations are divided into cancellations of limit orders at the best (blue filled diamond) and cancellations of limit orders inside the book at the time when they are canceled (orange open diamond). The data refer to the stock AZN.

and there are also varying degrees of asymmetry between positive and negative Δm_0 fluctuations. As seen in the inset in Fig. 6, the average over the first 55 stocks shows again a clear piecewise linear behavior. When $\Delta m_0=1/2$ ($\Delta m_0=-1/2$) tick the midprice permanent impact is on average 0.17 (−0.25) ticks. This again shows the presence of a trending phase of the midprice, which reinforces the direction of the price change. For larger price change there is on the contrary a partial reversion of the price. For positive fluctuations the Δm_0 linear fit gives the relation $\langle R_m(\Delta m_0)\rangle=0.45-0.46\Delta m_0$, while for the negative range we obtain $\langle R_m(\Delta m_0)\rangle=-0.62-0.52\Delta m_0$. The Δm_0 independent part is 0.45 ticks for positive values of Δm_0, whereas for negative values it is significantly larger, −0.62 ticks. In other words negative fluctuations have a larger knock-on effect than positive ones even for small fluctuations. The total fluctuation including the initial impact, $\langle I_m(\Delta m_0)\rangle$, is shown in Fig. 7. The permanent impact has a behavior intermediate between the zero impact assumption and the completely permanent impact assumption. From a linear fit of the curve for positive and negative values of Δm_0 we obtain $\langle I_m(\Delta m_0)\rangle=0.45+0.54\Delta m_0$ (positive values) and $\langle I_m(\Delta m_0)\rangle=-0.62+0.48\Delta m_0$ (negative values).

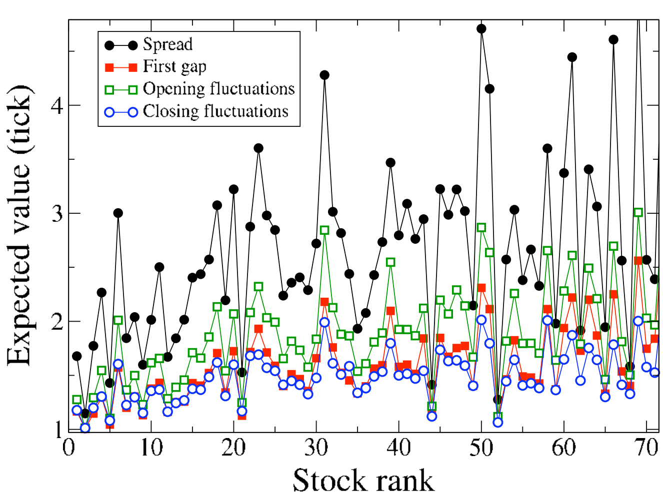

FIG. 8. (Color online) Mean value of spread (black filled circle), first gap (red filled square), positive (green open square), and negative (blue open circle) spread fluctuations for 71 stocks in our database. All the quantities are expressed in ticks and the stocks are ordered by size of database. For the definition of these quantities see the text.

In conclusion the permanent part of the fluctuation impact is roughly linear in the price (ask or mid) that is used as a conditioning variable. The midprice permanent impact is roughly symmetric for positive and negative fluctuations. The ask permanent impact instead shows a clearly asymmetric profile. The ask permanent impact $\langle I_a(\Delta a_0\ \rangle$ conditional to a given positive ask fluctuation Δa_0 at the initial time is in absolute value smaller than the ask permanent impact conditional to a negative initial ask fluctuation $-\Delta a_0$. This asymmetry is a consequence of the different causes for positive and negative ask fluctuations, as well as of the mean reverting and positivity property of spread.

V. EMPIRICAL EVIDENCE OF STRATEGIC LIMIT ORDER PLACEMENT

In this section we present empirical evidence supporting the hypothesis that the power-law decay of the conditional spread is a manifestation of the strategic placement of limit orders, market orders, and cancellations.

Let us start with an overview of how the spread and related quantities vary across the 71 stocks in our database, which is shown in Fig. 8. All quantities are sampled every

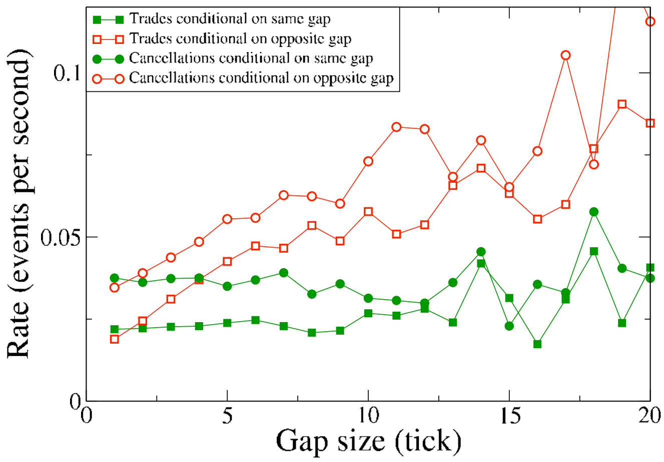

FIG. 10. (Color online) Rate of transactions (open and filled squares) and cancellations (open and filled circles) conditional to the size of the same (green filled symbols) or opposite (red open symbols) gap. The data refer to the stock AZN.

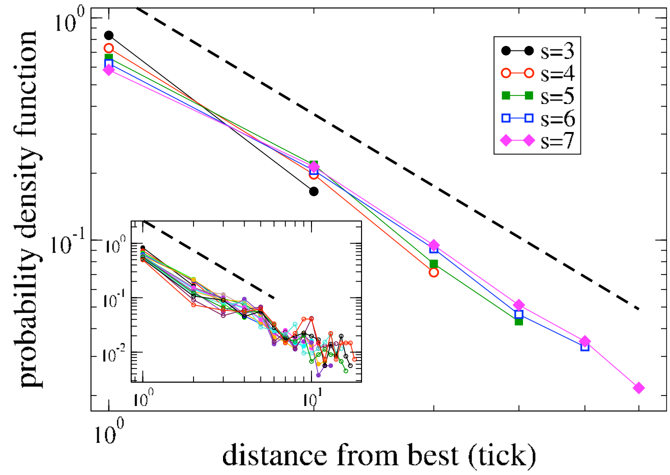

FIG. 11. (Color online) Probability density function of the distance between price of limit order in the spread and the best available price (ask for sell limit orders and bid for buy limit orders) for various values of the spread s: $(s=3)$ (black filled circle), $(s=4)$ (red open circle), $(s=5)$ (green filled square), $(s=6)$ (blue open square), and $(s=7)$ (purple filled diamond). The dashed line is a power law decaying function with exponent of 1.8. The inset shows more values of the spread size s up to $s=19$. The data refer to the stock AZN.

second. Specifically, we compute (i) the mean value of the spread $E[s(t)]$, (ii) the mean value of the (symmetrized) first gap $(1/2)E[a_2(t)-a(t)]+(1/2)E[b(t)-b_2(t)]$, where $a_2(t)$ and $b_2(t)$ denote the second ask price level and second bid price level, respectively, and (iii) the mean value of what we call "opening (closing) fluctuations." The mean value of opening fluctuations is obtained by considering $E[a(t+1)-a(t)|a(t+1)>a(t)]$ and $E[b(t)-b(t+1)|b(t+1)<b(t)]$ with analogous expressions for the closing fluctuations. For spread opening fluctuations and spread closing fluctuations, the mean value is taken given that there is a fluctuation in either the bid or ask or both. Finally in the figure the stocks are ordered by size of database. The database size roughly corresponds to activity occurring in each stock during the year 2002, where high activity means a high order submission rate, a high trading rate, etc. This therefore suggests that, in general, the spread, and spread related quantities, decreases with increasing activity. Figure 8 shows that spread closing fluctuations are smaller in size than spread opening fluctuations. This means that spread closing fluctuations must be more numerous than spread opening fluctuations to maintain a stationary spread. This is therefore the first supporting evidence of the strategic submission of limit orders after a spread opening.

The dynamics of the spread is determined by the flow of market orders, limit orders falling in the spread, and cancellations of the orders at the best bid and ask. The rates of the three different types of orders strongly depend on the spread size. Figure 9 shows the rates (in events per second) of different possible events in the limit order book, specifically trades (market orders), limit orders, and cancellations. Limit orders are divided in three subsets according to their limit price. We consider limit orders in the spread, limit orders at an existing best (bid or ask), and limit orders placed inside the book. Similarly cancellations are divided into cancellations of limit orders at the best and cancellations of limit orders inside the book (at the time when they are canceled). Figure 9 shows that the rate of trading decreases as the spread increases, whereas the rate of limit order submission in the spread dramatically increases with spread size. A related decaying behavior of the probability that a submitted order is a market order conditional to the spread size has been observed by Mike and Farmer [26]. This behavior is intuitively expected since a large bid-ask spread is a strong disincentive to trade, given that the spread related cost is large. On the other hand, a large spread is an incentive for limit order placement inside the spread in order to have priority of execution at a convenient price. Also the cancellation rate increases with spread. These findings are consistent with a process whereby liquidity providers cancel and replace limit orders in order to slowly close the spread.

The increase in the rate of limit order submission and cancellation at the best and the decrease in market order rate are also consistent with the hypothesis of strategic placement of submitted orders and cancellations. This scenario is also consistent with models and empirical observations assuming that spread dynamics reflects the difficulties that uninformed traders face due to the fact that information carried by trading events is not certain and suggesting that the intervals between trades may have information content which is not exogenous to the price process [5,10].

Beside the spread and its fluctuations, another important quantity determining liquidity is the gap size. As described above the gap size is the absolute price difference between the best available price (e.g., to buy) and the next best available price. Gap size is important because it has been suggested that immediate price impact is strongly determined by gap size [29]. In any given instant there are two gaps, one on the buy side and one on the sell side of the limit order book. For a buy (sell) market order we define the same side gap size as the gap size on the buy (sell) limit order book side and opposite side gap size as the gap size on the sell (buy) limit order book side. In Fig. 10 we show the trade rate

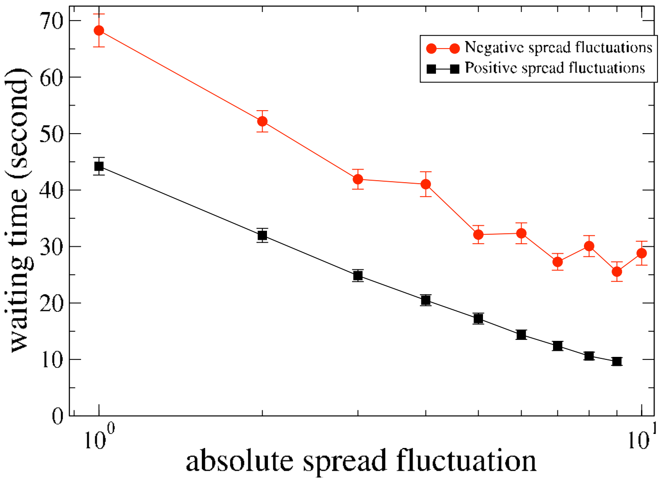

FIG. 12. (Color online) Mean waiting time between a spread variation $\Delta = s(t) - s(t-1)$ and the next spread changing event as a function of $|\Delta|$ for positive (black filled square) and negative (red filled circle) spread fluctuations. The figure shows an average of the mean waiting time across the stocks of our sample.

conditional to the same and to the opposite gap size. We see that while the trade rate is almost independent of same side gap size, the rate increases significantly with the opposite side gap size. A possible interpretation of this result is the following. Imagine the market is drifting upward—i.e., the price is increasing. Then most trades are buy initiated and the gap on the sell side is large. Buy limit orders tend to be submitted just inside the spread, i.e., beating the best buy by one tick, so the gap on the buy side is small.

A similar result is seen for cancellations. Figure 10 also shows the cancellation rate conditional to the size of the same and that of the opposite gap. Again it is seen that the cancellation rate weakly depends on the same side gap size, whereas it increases with the size of the opposite gap. When the price is drifting, for example, upward, there is a strong limit order flow and cancellation on the buy side of the book. As described above this might be due to the fact that liquidity providers try to gain the best bid by placing buy limit orders in the spread and canceling beaten buy limit orders to get a better position.

The way in which limit orders are placed into the spread when the spread is large is also worth investigating. Limit order placement in the spread follows an interesting scaling relation originally observed by Mike and Farmer [26]. In Fig. 11 we show the distribution of the distance from the same best of limit orders placed in the spread for different values of the spread. Specifically, given a spread size $s(t) = a(t) - b(t)$ and a limit order with price p between the ask $a(t)$ and the sell $b(t)$, we consider the distribution of $a(t) - p$ for sell limit orders and $p - b(t)$ for buy limit orders. The shape of the curves shown in Fig. 11 is consistent with a power-law decay with an exponent of ~ 1.8. Mike and Farmer [26] fitted the limit order placement with a Student's t-distribution with 1.3 degrees of freedom. This distribution indicates that when the spread is large, limit orders are not placed simply in a way that immediately reverts the spread back to its typical (small) value. Rather limit orders are sequentially placed close to the existing best price and this might lead to a slow decay of the spread.

In addition to the investigation of the rate of orders as a function of the size of the spread (Fig. 9) one can also measure the time interval between a spread variation of a given size and the next spread variation (of any size). This waiting time gives the reaction time of the market to an abrupt spread variation. In Fig. 12 we show the mean waiting time between a spread variation $\Delta = s(t) - s(t-1)$ and the next spread variation as a function of Δ. The waiting time decreases when the spread variation increases and the functional dependence is approximately logarithmic. In other words the larger the spread variation, the shorter the time one has to wait until a new event changes the spread again. Moreover the waiting time for positive spread variations is much shorter than the waiting time for negative spread variations of the same size (in absolute value). This result is to be expected and shows that the market reacts faster to an increase in the spread rather than to a decrease in the spread.

In summary we believe that we have provided empirical evidence supporting the hypothesis that the power-law slow decay of the spread could be a manifestation of the presence of a multiplicity of time scales introduced by the heterogeneity of traders and/or by the heterogeneity of different strategies of order submissions and cancellations used by them.

VI. DISCUSSION

In this paper, we have shown some empirical facts of limit order book and price dynamics in double auction financial markets, specifically the slow scale-free power-law decay of the spread and the approximately linear relation between permanent and immediate market impacts triggered by an order book change affecting the ask or the midprice.

The power-law spread decay occurring after a sudden opening of the bid-ask spread is consistent with the hypothesis of strategic placement of limit orders inside the spread. These strategic limit order submission procedures are most probably performed to attain execution priority at the best ask or bid price after temporary liquidity crises.

Economic theory suggests that the spread opening can also be seen to be the result of increased uncertainty about the information processed by the market makers [5,10]. From this perspective the limit order submission by market makers occurring after an abrupt change in the spread could be seen to be a part of a learning procedure devoted to the discovery of the most appropriate levels for the bid and the ask prices. We observe that the dynamics of the bid-ask spread presents a power-law decay to its long term average value. The power-law decay is a functional form without a characteristic time scale and this property is consistent with a scenario describing this process as a learning process. In fact, learning processes are quite often characterized by a power-law optimization profile.

However, the presence of a power-law decay to the normal value of the spread is a necessary but not sufficient condition to ensure strategic placement of limit orders. In fact, power-law decay has also been observed in zero intelligence deposition models of order flow after an exogenous abrupt spread opening [36]. However Toth *et al.* [36] stated that their zero intelligence model does not explain all the spread reversion dynamics. In addition to the power-law decay of the spread we also observe variation in the statistical properties of order flow of limit and market orders as a function of the size of the spread. We believe that these findings support the hypothesis of strategic placement of limit and market orders after an abrupt opening of the spread.

The second focus of our paper is on the permanent price impact induced by any event altering the spread. Our investigation shows that the permanent impact is statistically detectable and provides relevant information for the modeling of price formation in high-frequency data both on the ask and on the midprice. We observe that the permanent parts of the ask and midprice fluctuation are approximately linear functions of the immediate fluctuation impact.

This proportionality could be important in the search for the origin of fat tails in price changes. In Ref. [29] it has been shown that the distribution of nonzero immediate impacts Δm_0 matches the distribution of first gaps very well. This suggests that a major determinant of the origin of large price changes is the presence of large gaps in the limit order book when the market is in a state of lack of liquidity and/or uncertainty about incoming information. Clearly the correspondence observed in [29] holds only for individual returns (impact) and it is not *a priori* obvious that one can extend it to longer time scales. The results in Fig. 7 give support to the idea that temporary fluctuations of the order book are also responsible for the fat tails of price changes at longer time scales. In fact, the distribution of gaps is equal to the distribution of immediate impacts and Fig. 7 shows that permanent impact is a linear function of immediate impact.

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